# Pressure- and Temperature-Dependence of the Electrical Resistivity of Some Nonmagnetic d-electron Amorphous Alloys

G. Fritsch\*, J. Willer, and E. Lüscher

Physik Department, Technische Universität München, 8046 Garching

Z. Naturforsch. 37a, 1235-1240 (1982); received June 23, 1982

Using the Ziman-formulation a model is derived which describes the pressure dependence of the electrical resistivity of d-electron amorphous alloys. It is shown how the volume dependence of the scattering-potential, electron-gas-properties and the Grüneisenparameter determine the P-dependence of a constant and a temperature dependent term in the resistivity. The absolute value of the resistivity is discussed.

#### 1. Introduction

The temperature dependence of the electrical resistivity at temperature T above the Debye-temperature  $\theta$  can successfully be described by the Ziman-equation, if the mean free path of the electrons is larger than the atomic distances [1]. At temperatures  $T < \theta$ , this equation has to be modified, i.e. the static structure factor for the inelastic part should be replaced by the dynamic one. This formulation is called the Baym-equation [2].

The static structure factor contains contributions from elastic and inelastic scattering processes. However, for d-electron amorphous alloys, the main contributions to the integral over the wave-vector q in the Ziman expression for  $\varrho$  originates from the elastic part. This is shown later in Sect. 2 and 4. Whereas the elastic part doesn't change dramatically with temperature, the inelastic part decreases in magnitude with decreasing temperature. For this reason, we assume that the Ziman equation holds down to temperatures of at least  $\theta/2$  [2].

Since the mean free path is only several atomic distances, there is some discussion in literature [3], if the Ziman-formulation can give a correct description of the situation. In this paper we would like to check this point by comparing theoretical predictions of the P- and T-dependence of the electrical resistivity with experimental results.

The paper is organized as follows, at first a simple model [4] is derived, based on an effective amor-

ZWE Physik, Hochschule der Bundeswehr München,

phous "element" [5] and then contact is made with experimental results for several amorphous alloys.

# 2. The Temperature and Volume Dependence of the Electrical Resistivity

For a single component system the electrical resistivity can be written in the Ziman formulation as [1]:

$$\varrho = 3 \pi m^2 \Omega / (\hbar^3 e^2 k_F^2 z) 
\cdot \int_0^1 |v(x)|^2 S(x) 4 x^3 dx,$$
(1)

where  $x = q/(2 k_{\rm F})$  with  $k_{\rm F}$  the Fermi-vector, v(x) is the electron-ion pseudo potential, S(x) the static structure factor, e, m the electronic charge and mass, respectively, z the number of free electrons per ion and  $\Omega$  the atomic volume.

In case of systems containing d-electrons it is necessary to replace the pseudo-potential by the *t*-matrix [6], i.e.

$$egin{aligned} v(q) &
ightarrow t(q, E_{\mathbf{F}}) \ &= -2\,\pi\,\hbar^3\,z/[\varOmega\,m\,(2\,m\,E_{\mathbf{F}})^{1/2}] \ &\cdot \sum\limits_{l=0}^{\infty} (2\,l+1)\sin\,\delta_l(E_{\mathbf{F}})\exp\left[i\,\delta_l(E_{\mathbf{F}})
ight] \ &\cdot P_l[\cos\,\vartheta(q)]\,. \end{aligned}$$

where  $E_{\rm F}$  denotes the Fermi-energy,  $\delta_l$  is the scattering phase shift for the lth partial wave and  $\sin \left[ \frac{\partial (q)}{2} \right] = x$ .

Substituting (2) into (1) yields

$$\varrho = A \int_{0}^{1} |\Sigma_{l}(x)|^{2} S(x) 4 x^{3} dx, \qquad (3)$$

where  $A = 6 \pi^3 \hbar^3 z / (e^2 m k_{\rm F}^2 E_{\rm F} \Omega)$  and  $\Sigma_l(x)$  is an abbreviation for the sum in (2). If one considers the

D-8014 Neubiberg.

Reprint requests to Prof. Dr. G. Fritsch, ZWE Physik,

Hochschule der Bundeswehr München, Werner Heisenberg Weg, D-8014 Neubiberg.

0340-4811 / 82 / 1100-1235  $\$  01.30/0. — Please order a reprint rather than making your own copy.



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung "Keine Bearbeitung") beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermödlichen

T-dependence of  $\varrho$ , then the only relevant contribution comes from the static structure factor S(x), since the other quantities depend only weakly on T via the thermal volume expansion. This effect may be safely neglected. Such a conclusion is no longer valid when varying the pressure since all factors in (3) depend on the volume, i.e. A,  $\Sigma_l(x)$  as well as S(x). Whereas the T-dependence has been widely discussed in literature [1, 2], the V-dependence has not. Therefore, we would like to present this analysis for small relative volume changes  $\Delta V/V \ll 1$ .

The static structure factor S(x) can be written to a good approximation as [7]

$$S(x) = 1 + [S_0(x) - 1] \cdot \exp[-2W(T, V, x)].$$
(4)

Here  $S_0(x)$  is the structure factor defined by the atomic equilibrium positions. It is assumed that the structure does not change under pressure, except for a change of length scale. Under these assumptions  $S_0(x)$  does not vary with V, since x is independent of this length scale.

2W(T, V, x) denotes the Debye-Waller exponent. For  $T > \theta/2$ , it is

$$2W(T, V, x) = B(V, T)x^{2}T$$
 (5)

with  $B = 12 h^2 k_F^2(V) / [M k_B \theta^2(T, V)]$ .

Here M is the atomic mass and  $\theta(V, T)$  the Debyetemperature.  $\theta(V, T)$  can be defined by fitting the specific heat to the Debye-expression. For amorphous metallic systems  $\theta$  should depend on temperature explicitly. The reason is that in these systems the transverse modes [8] seem to be appreciably softer than in the crystalline case. At low temperature these modes should dominate the specific heat and hence produce a lower  $\theta$ . We denote the high temperature limit by  $\theta_{\rm H}$ . In addition one should keep in mind, that (5) is the high temperature approximation valid for  $T > \theta_{\rm H}/2$ . At lower temperatures one should expect a more complicated behaviour [7]. The Debye-Waller factor can be expanded in most cases, since  $2W(T, V, x) < 5 \cdot 10^{-2}$ . Using this approximation together with (4) and (5), (3) may be written as

$$\rho(T, V) = \rho_0(V) + \rho_T(V, T) \tag{6}$$

with

$$\varrho_0(V) = A(V) \int_0^1 |\Sigma_l(x, V)|^2 S_0(x) 4 x^3 dx$$
 (7)

and

$$\varrho_{T}(V,T) = -A(V)B(V,T)T$$

$$\cdot \int_{0}^{1} |\Sigma_{l}(x,V)|^{2} [S_{0}(x)-1] 4x^{5} dx.$$
(8)

The explicit T- and V-dependences of the quantities A, B and  $\Sigma_l(x)$  have been indicated in (7) and (8). At higher temperatures  $T > \theta_H$ , B(V, T) should become independent of T, since  $\theta(V, T) \to \theta_H(V)$ . In this case it follows that  $\varrho_T(V, T) = \hat{\varrho}_T(V) T$ .

#### 3. The Pressure Dependence of the Resistivity

Now let us consider the volume dependence of (7) and (8) in the limits  $(V-V_0)/V_0 = \Delta V/V_0 \ll 1$ . We will use a formulation for  $E_{\rm F}(V)$  which will allow to incorporate the effects of d-electrons, i.e. we take

$$E_{\mathbf{F}}(V) = \hbar^2 k_{\mathbf{F}}^2(V) / \{2 \, m \, \lceil k_{\mathbf{F}}(V), \, V \rceil \}.$$

Furthermore, applying  $k_{\rm F}(V) = (3 \pi^2 z N_0/V)^{1/3}$  with  $N_0$  Avogadro's number, we get from (7):

$$\varrho_{0}(V) = A_{0} \int_{0}^{1} |\mathcal{L}_{l}(x, V_{0})|^{2} S_{0}(x) 4 x^{3}$$

$$\cdot [1 + \sigma_{0}(x) (\Delta V/V_{0})] dx$$
 (9)

with

$$\sigma_0(x) = \frac{1}{3} + 2 \Delta_1^{\text{eff}}(x),$$
 (10)

where

$$2 \mathcal{L}_{1}^{\text{eff}}(x) = 2 \mathcal{L}_{l}(x) - (\beta + \frac{2}{3}),$$

$$\mathcal{L}_{l}(x) = (|\mathcal{L}_{l}(x, V_{0})|)^{-1}$$

$$\cdot [d |\mathcal{L}_{l}(x, V)|/d \ln V]_{V = V_{0}},$$

$$\beta = [d \ln (m E_{F})/d \ln V]_{V = V_{0}},$$
(11)

and

$$A_0 = 6 \pi^3 \hbar^3 z / (e^2 m k_{F0}^2 E_{F0} \Omega_0). \tag{12}$$

The subscript zero denotes quantities at P=0. In case that  $m[k_{\rm F}(V), V]$  does not depend on  $k_{\rm F}(V)$ , we will get the free electron value for  $\beta$ , which is -2/3 and consequently  $\Delta_1^{\rm eff}(x) = \Delta_1(x)$ .

In the same way we calculate from (8):

$$\varrho_{T}(V,T) = -A_{0}B_{0}(T)T$$

$$\cdot \int_{0}^{1} |\Sigma_{l}(x,V_{0})|^{2} [S_{0}(x) - 1] 4x^{5}$$

$$\cdot [1 + \sigma_{T}(x)(\Delta V/V_{0})] dx \quad (13)$$

with

$$\sigma_T(x) = 2\gamma_G - \frac{1}{3} + 2\Delta_I^{\text{eff}}(x)$$
. (14)

Here  $\gamma_G$  denotes the Grüneisenparameter, defined as  $\gamma_G = -(\partial \ln \theta(V, T)/\partial \ln V)_{T, V_0}$ . In this approximation we consider  $\gamma_G$  to be a constant.

$$B_0(T) = 12 \,\hbar^2 \,k_{\rm F0}^2 / [M \,k_{\rm B} \,\theta_0^2(T)] \,. \tag{15}$$

The pressure dependence is easily calculated by taking

$$\Delta V/V_0 = - \varkappa P, \tag{16}$$

where  $\varkappa$  denotes the isothermal compressibility.

# 4. Approximation for d-Electrons System

Next, we would like to introduce a simplification by assuming that the resistivity is dominated by the scattering of d-electrons (l=2). The d-contribution to the t-matrix appears mainly at  $x\approx 1$ , because the quantity  $P_2[\cos\vartheta(q)]$  in (2) varies with x according to

$$P_2(x) = 1 - 6x^2 + 6x^4$$
.

Considering the function  $S_0(x)$ , which peaks at x=1, and the factors  $x^3$  and  $x^5$  in the integrals, respectively, one can replace  $S_0(x)$  by  $S_0(1) = S_0(2 k_{\text{F0}})$  and  $[\Sigma_2(x, V_0)]^2$  by

$$[\Sigma_2(1, V_0)]^2 = 25 \sin^2 \delta_2(E_{\rm F0}, V_0)$$
.

This is also done in the parameters

$$\Delta_l(x) \approx \Delta_l(1) = \Delta_l$$

(see (9) and (11)). In the case of d-electrons, this yields

$$\Delta_2 = \operatorname{ctg} \delta_2(E_{\mathbf{F}0}, V_0) \cdot [\operatorname{d}\delta_2(E_{\mathbf{F}}, V)/\operatorname{d} \ln V]_{V=V_0}. \tag{17}$$

With these approximations we derive from (9)

$$\varrho_0(V) = \varrho_{00}[1 + \sigma_0(\Delta V/V_0)] 
= \varrho_{00}[1 - \alpha_0 P],$$
(18)

and similarly from (13)

$$\varrho_T(V,T) = \varrho_{T0}(T) \left[ 1 + \sigma_T(\Delta V/V_0) \right]$$

$$= \varrho_{T0}(T) \left[ 1 - \alpha_T P \right]$$
(19)

with

$$\varrho_{00} = 5 A_0 \sin^2 \delta_2(E_{F0}, V_0) 
\cdot S_0(2 k_{F0})$$
(20)

and

$$\begin{split} \varrho_{T0}(T) &= -\frac{80}{21} A_0 \, B_0(T) \sin^2 \delta_2(E_{\rm F0}, \, V_0) \\ & \cdot \left[ S_0(2 \, k_{\rm F0}) - 1 \right] T \\ &= \hat{\varrho}_{T0} \, T \,, \\ &\text{if } B_0(T) \text{ is independent of } T \,, \end{split} \tag{21}$$

where (see (10), (14) and (16))

$$\sigma_0 = \frac{1}{3} + 2 \Delta_2^{\text{eff}},$$

$$\alpha_0 = (\frac{1}{3} + 2 \Delta_2^{\text{eff}}) \varkappa \quad \text{and}$$
(22)

$$\sigma_T = 2\gamma_G - \frac{1}{3} + 2\Delta_2^{\text{eff}},$$
 $\alpha_T = (2\gamma_G - \frac{1}{2} + 2\Delta_2^{\text{eff}})\varkappa.$  (23)

## 5. Experimental Data

For binary or ternary alloys [6] (1) has to be extended in order to incorporate the partial structure factors and the various pseudopotentials. However, for a qualitative analysis we would like to ignore this complication and replace the amorphous alloy by an artificial "element", based on average quantities. In this case the term  $v(x)^2S(x)$  in the Ziman integral should read

$$[c_1 c_2 (v_1(x) - v_2(x))^2 + (c_1 v_1(x) + c_2 v_2(x))^2 S(x)],$$
 (24)

where the  $c_i$ 's are the concentrations and the  $v_i$ 's the pseudopotentials of the two components. An estimation of this expression for  $v_1(x) \approx v_2(x) \approx v(x)$  yields  $v^2(x)S(x)$ , and for the case  $v(x) = v_1(x) \geqslant v_2(x)$  it gives roughly  $0.5 \ v^2(x)S(x)$ . In the latter case it was assumed that  $c_1 = 0.7$  and  $S(x) \approx 2$  (d-electron-case). Hence, we estimate that this procedure introduces an uncertainty in the absolute magnitude of the resistivity of not more than a factor of two.

We would like to compare the experimental results obtained for the amorphous d-electron alloys Ti<sub>50</sub>Be<sub>40</sub>Zr<sub>10</sub> [10], Pd<sub>30</sub>Zr<sub>70</sub> [9], Cu<sub>40</sub>Zr<sub>60</sub> [9] and Cu<sub>57</sub>Zr<sub>43</sub> [10] with the predictions derived from the model outlined above. A least squares fit to the date according to

$$\varrho(P,T) = \varrho_0(P) + \varrho_1(P)T + \varrho_2(P)T^{3/2}$$
 (25)

proved to be a good phenomenological description of the data. By introducing the notation

$$\varrho_i(P) = \varrho_{i0}(1 - \alpha_i P) \tag{26}$$

for i=0 to 2, the results given in Table 1 can be derived. In view of accuracy of the data

$$(\Delta \varrho/\varrho \approx 10^{-5}, \quad \Delta P/P \approx 5 \cdot 10^{-2})$$
 [9]

only a linear P-dependence has been assumed.

#### 6. Discussion

# a) T-dependence for P=0

The temperature-dependence of the resistivity is described by (19). Within the model given here the

Table 1. Results from a fit of (25) and (26) to the data. The  $Ti_{50}Be_{40}Zr_{10}$  sample (I) corresponds to thickness of 35  $\mu m$ , the sample (II) to 40  $\mu m$  ( $\pm 2$   $\mu m$ ). ([a] For literature see [10].) The error bars given do not include the uncertainties in the geometrical factors of about 10%.

Alloy	$\mathrm{Pd}_{30}\mathrm{Zr}_{70}$	$\mathrm{Cu_{40}Zr_{60}}$	$\mathrm{Cu}_{57}\mathrm{Zr}_{43}$	${ m Ti}_{50}{ m Be}_{40}{ m Zr}_{10}$ (I)	${ m Ti}_{50}{ m Be}_{40}{ m Zr}_{10}$ (II)
$\begin{array}{c} \hline \\ \varrho_{00}/\mu\Omega \ cm \\ \varrho_{10}/n\Omega \ cm \ K^{-1} \\ \varrho_{20}/n\Omega \ cm \ K^{-3/2} \\ \alpha_0/GPa^{-1} \cdot 10^2 \\ \alpha_1/GPa^{-1} \cdot 10^2 \\ \alpha_2/GPa^{-1} \cdot 10^2 \\ \varkappa/m^3 \ J^{-1} \cdot 10^{11} \end{array}$	$\begin{array}{c} 148 \pm 2 \\ - (57 \pm 2) \\ 1.7 \pm 0.1 \\ 1.05 \pm 0.07 \\ 3.2 \pm 0.1 \\ 3.1 \pm 0.4 \\ 1.0 \ \ [\mathrm{a}] \end{array}$	$180\pm2 \ -(60\pm10) \ 1.8\pm0.1 \ 1.3\pm0.1 \ 2.9\pm1.2 \ 3.0\pm0.7 \ 1.0[a]$	$egin{array}{c} 224\pm2 \\ -(51\pm1) \\ 1.20\pm0.05 \\ 0.73\pm0.09 \\ 2.9\pm0.2 \\ 4.8\pm0.5 \\ 0.96 [\mathrm{a}] \\ \end{array}$	$247 \pm 5 \ - (103 \pm 3) \ 2.8 \pm 0.2 \ 0.9 \pm 0.1 \ 2.7 \pm 0.2 \ 3.6 \pm 0.3 \ 1.02  [\mathrm{a}]$	$egin{array}{c} 246\pm5 \ -\ (95\pm3) \ 2.6\pm0.1 \ 0.95\pm0.1 \ 2.4\pm0.5 \ 2.8\pm0.3 \ 1.02 \mathrm{[a]} \end{array}$

relevant factors originate from the Debye-Waller exponent. With the aid of (15) and (21) we may write

$$\varrho_T(V_0, T) \sim -T/\theta_0^2(T)$$
 (27)

for  $T > \theta_{\rm H0}/2$ . The index 0 indicates P = 0.

As was already discussed, the Debye-temperature  $\theta(T)$  should decrease with decreasing temperature T. However, at higher temperatures  $T > \theta_{\rm H0}$ , it should become a constant. These facts would imply an increasing slope of the linear term in T with decreasing temperature and a constant slope for  $T > \theta_{\rm H0}$ . Qualitatively such a behaviour is observed, at least at lower temperatures. However, quantitatively the Debye-temperature  $\theta(T)$  is not a constant for  $T > \theta_{\rm H0}$ : This behaviour is shown in Fig. 1, where  $\theta_0(T)$  was determined as follows. From (25) we have in case of P = 0:

$$\varrho(0, T) = \varrho_{00} + \varrho_{10} \cdot [1 + (\varrho_{20}/\varrho_{10}) T^{1/2}] T,$$
 (28)

where the notations introduced by (26) have been

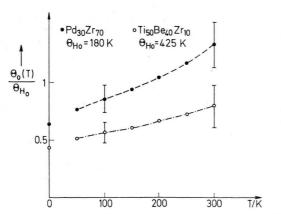


Fig. 1. Normalized Debye-temperature  $\theta_0(T)/\theta_{\rm H0}$  at P=0 as a function of temperature T for the alloys  ${\rm Pd_{30}Zr_{70}}$  and  ${\rm Ti_{50}Be_{40}Zr_{10}}$ : The lines are drawn as a guide to the eye.

used. The numerical values of the quantities  $\varrho_{i0}$  are given in Table 1.

After comparison with (27) and proper normalization, (28) yields

$$\Theta_0(T)/\Theta_{H0} = \{ [1 + (\varrho_{20}/\varrho_{10}) \Theta_{H0}^{1/2}] /$$

$$[1 + (\varrho_{20}/\varrho_{10}) T^{1/2}] \}^{1/2}.$$
(29)

This function has a pole at

$$T = (-\varrho_{10}/\varrho_{20})^2 > 10^3 \, \mathrm{K}$$
 .

It shows parabolic behaviour at lower temperatures. The ratio  $\rho_{20}/\rho_{10}$  is always negative.

Since, above  $\Theta_{\rm H0}$ , the Debye-temperature  $\Theta_0(T)$  doesn't converge towards a limiting value, there should be additional temperature dependent terms, if the Ziman-model is correct. These terms are not included in the simple formulation of (4) and (5).

However, when discussing the total value of the resistivity below, we need the prefactor of the linear term in T, i.e.  $\hat{\varrho}_{T0} = [\mathrm{d}\varrho_T(V,T)/\mathrm{d}T]_{V=V_0}$ . In view of the behaviour described above, we decided to define this quantity as the derivative of (25) for  $B_0 = \mathrm{const}$ , taken at  $T = \Theta_{\mathrm{H}0}$ . Hence, we may write

$$\hat{\varrho}_{T0} = [\mathrm{d}\varrho_{T}(V,T)/\mathrm{d}T]_{\substack{T = \Theta_{\mathrm{H0}} \\ V = V_{0}}} 
= \varrho_{10} + \frac{3}{2}\,\varrho_{20}\,\Theta_{\mathrm{H0}}^{1/2}.$$
(30)

In order to check this assumption, we evaluated  $S_0(2 k_{\rm F0})$  from the ratio of (20) and (21). The results are given in Table 2. The numbers used in the calculations are summarized in Table 3. The data for the  ${\rm Ti}_{50}{\rm Be}_{40}{\rm Zr}_{10}$ -alloy have been extrapolated with the aid of (25), since  $\Theta_{\rm H0}=425~{\rm K}$  is above the temperature range examined. The results for  $S_0(2 k_{\rm F0})$  are not unreasonable, being close to two. In view of the uncertainties involved, the situation  $P \neq 0$  is not considered here.

Table 2. Results for the amorphous alloys. The  $Ti_{50}Be_{40}Zr_{10}$  sample (I) corresponds to a thickness of 35  $\mu$ m, the sample (II) to 40  $\mu$ m ( $\pm$  2  $\mu$ m).

Alloy	$\mathrm{Pd}_{30}\mathrm{Zr}_{70}$	$\mathrm{Cu_{40}Zr_{60}}$	$\mathrm{Cu}_{57}\mathrm{Zr}_{43}$	${\rm Ti}_{50}{\rm Be}_{40}{\rm Zr}_{10}\ ({\rm I})$	$\mathrm{Ti_{50}Be_{40}Zr_{10}}~\mathrm{(II)}$
$egin{aligned} -\frac{\mathrm{d}arrho/\mathrm{d}T/_{ heta_{ m H0}}}{S_0(2k_{ m F0})}\mathrm{n}\Omega\mathrm{cm}\mathrm{K}^{-1}\ S_0\left(2k_{ m F0} ight)\ \mathcal{A}_2^{ m eff}\ \mathcal{A}_2^{ m G}\ \sin^2\delta_2(E_{ m F0},V_0) \end{aligned}$	$23\pm3 \ 2.2\pm0.4 \ 0.4\pm0.1 \ 1.4\pm0.1 \ 3.7\pm0.4 \ 0.012$	$24\pm10 \ 1.7\pm0.7 \ 0.5\pm0.1 \ 1.2\pm0.4 \ 2.2\pm0.3 \ 0.025$	$\begin{array}{c} 22.8 \ \pm 1.6 \\ 2.1 \ \pm 0.3 \\ 0.21 \pm 0.05 \\ 1.4 \ \pm 0.2 \\ 3.1 \ \pm 0.1 \\ 0.051 \end{array}$	$\begin{array}{c} 16\pm7 \\ 2.0\pm0.9 \\ 0.3\pm0.1 \\ 1.2\pm0.2 \\ 2.9\pm0.1 \\ 0.036 \end{array}$	$21 \pm 4$ $2.1 \pm 0.4$ $0.3 \pm 0.1$ $1.1 \pm 0.2$

# b) P-dependence

If the compressibilities are known (see Table 1), the pressure dependence of the data should be governed by the two parameters  $\Delta_2^{\text{eff}}$  and  $\gamma_G$  as given by (22) and (23). According to these equations we have for the phaseshift parameter

$$\Delta_2^{\text{eff}} = (\alpha_0/\kappa - \frac{1}{3})/2 \tag{31}$$

and for the Grüneisen parameter

$$\gamma_{\rm G} = (\alpha_T - \alpha_0)/(2\varkappa) + \frac{1}{3}$$
. (32)

Equation (32) is independent of  $\varDelta_2^{\rm eff}$ . Since  $\alpha_1 \approx \alpha_2$  in most cases, we use  $\alpha_1 = \alpha_T$ . The results for  $\varDelta_2^{\rm eff}$  and  $\varUpsilon_G$  are also summarized in Table 2. The values obtained for  $\varUpsilon_G$  turn out to be between one and two. An analysis of the  $T_c(P)$  properties of  $Pd_{30}Zr_{70}$  as well as  $Cu_{40}Zr_{60}$  yields  $\varUpsilon_G=1.4$  and 1.2, respectively [11], in good agreement with the numbers evaluated here.

Next, we would like to comment on the magnitude of the parameter  $\Delta_2$  as given by (17). Since  $\Delta_2^{\rm eff}$  contains the derivative of the product  $mE_{\rm F}$  with respect to the volume V,  $\Delta_2$  cannot be determined from the present analysis.

However,  $\beta = [\mathrm{d} \ln (mE_{\mathrm{F}})/\mathrm{d} \ln V)_{V_0}$  may be evaluated in the case of a nearly free electron-gas, where it takes the value -2/3. Hence we have  $\Delta_2 = \Delta_2^{\mathrm{eff}}$ . As  $\delta_2(E_{\mathrm{F0}}, V_0) < 1$  in the spirit of the Ziman formulation (see also Table 2 and below), inspection of Table 2 shows that  $[\mathrm{d}\delta_2/\mathrm{d} \ln V]_{V=V_0} > 0$ . Therefore  $[\mathrm{d}\delta_2/\mathrm{d}P]_{P=0}$  should be negative. This quantity depends on the position of the d-bands with respect

to the Fermi-energy. It is not known how these parameters will change with pressure. Thus, further discussion must await theoretical predictions.

# c) Total Value of the Resistivity at P = 0

According to (12) and (20) the constant part of the resistivity  $\varrho_{00}$  is given by

$$\rho_{00} = C(z/\Omega_0) \sin^2 \delta_2(E_{F0}, V_0) S_0(2k_{F0}) \quad (33)$$

with

$$C = 60 \,\pi^3 \,\hbar/(e^2 \,k_{\rm F0}^4)$$
.

In order to obtain a second equation for the determination of the parameters  $z/\Omega_0$  and  $\sin^2 \delta_2$ , we apply a relation between the electron-phonon coupling-constant  $\lambda$  and the T-dependence of the resistivity at high temperatures [12]:

$$\lambda = [e^2 \hbar z / (4 \pi m k_B \Omega_0)] (21/16) \cdot [S_0 (2 k_{F0}) - 1]^{-1} \hat{\varrho}_{T0}.$$
 (34)

Use this result and (21) for  $T > \theta_{\rm H}/2$  yields

$$\lambda = D(z/\Omega_0)^2 \sin^2 \delta_2(E_{\text{F0}}, V_0) \tag{35}$$

with

$$\begin{split} D &= 90 \, \pi^2 \, \hbar^6 / [m^2 \, M \, E_{\rm F0} (k_{\rm B} \, \Theta_{\rm H0})^2] \\ &= 180 \, \pi^2 \, \hbar^4 / [m \, M \, k_{\rm F0}^2 (k_{\rm B} \, \Theta_{\rm H0})^2] \, . \end{split}$$

Equations (33) and (35) define two conditions for the parameters  $\sin^2 \delta_2(E_{\rm F0}, V_0)$  and the electron density  $z/\Omega_0$ . In case of Pd<sub>30</sub>Zr<sub>70</sub> and Cu<sub>40</sub>Zr<sub>60</sub> the  $\lambda$ -values are known from an analysis of the pressuredependence of the superconducting transition tem-

Alloy  $Pd_{30}Zr_{70}$ Cu40Zr60 Cu<sub>57</sub>Zr<sub>43</sub>  $\mathrm{Ti}_{50}\mathrm{Be}_{40}\mathrm{Zr}_{10}$  $M/{
m kg} \cdot 10^{27} \ \Omega_0/{
m m}^3 \cdot 10^{30}$ 133 126 60.9 159 14.75 [a] 18.64 [a] 17.74 [a] 17.05 [a] 425 2455 [a] 4 [a] 0.24\* 25 [a] 13.8 [a]  $\theta_{\rm H0}/{\rm K}$ 180 180 [a] [a]  $k_{\mathrm{F0}}/\mathrm{nm^{-1}}$ [a] [a] 14 0.62 [a] 0.55 [a]

Table 3. Some characteristic values for the amorphous alloys considered. The  $\lambda$ -values indicated by \* are estimated as described in the text. ([a] For literature see [10].)

perature  $T_{\rm c}$  [11]. In the other cases they were estimated from McMillan's expression for  $T_c$  using  $T_{\rm e}$ 's around 0.5 K as well as  $\mu^* = 0.09$  [11]. The relevant data and the results are also given in Tables 2 and 3. The number of conduction electrons per ion z turns out to be between 2 and 4, if  $S_0(k_{\rm F0}) = 3$  is assumed. Using  $S_0(2k_{\rm F0}) = 2$  instead would reduce the numbers by a factor of 2/3, since z is proportional to  $S_0(2k_{\rm F0})$ .

## 7. Conclusion

It was shown that the Ziman-formulation of the electrical resistivity is able to describe the experimental results for amorphous d-electron alloys. This

[1] J. M. Ziman, Phil. Mag. 6, 1013 (1961). — Glassy Metals I. Topics in Applied Physics V. 46, Eds. H. J. Güntherodt and H. Beck, Springer-Verlag, Berlin 1981. H. S. Chen, Rep. Progr. Phys. 43, 353 (1980). S. R. Nagel, Phys. Rev. B16, 1694 (1977).

[2] G. Baym, Phys. Rev. 135, 1691 (1964). — K. Froböse and J. Jäckle, J. Phys. F7, 2331 (1977).

[3] P. B. Allen and B. Chakraborty, Phys. Rev. B23, 4815

[4] A preliminaring account of this work was published in: Physics of Solids under high Pressure, eds. J. S. Schilling and R. N. Shelton, North Holland Publ. Comp. Amsterdam 1981, p. 239.

[5] P. Wochner and J. Jäckle, Preprint 1982.

statement holds though the resistivities range between 140 und 250  $\mu\Omega$  cm. The temperature range considered is 1.4 K up to 300 K. The model gives qualitatively correct results down to 1.4 K, if superconductivity is taken into account and a temperature dependent Debye-temperature is postulated. However, this fact is not easy to understand since the approximations used should break down at least for  $T < \theta/2$ .

The pressure dependence of the constant resistivity term is governed by the volume dependence of the scattering potential and electron gas properties. The T-dependent terms contain in addition the Grüneisen parameter, describing the changes in structure due to thermal vibrations.

- [6] R. Evans, D. A. Greenwood, and P. Lloyd, Phys. Letters 35A, 57 (1971).
- S. R. Nagel, Phys. Rev. B16, 1694 (1977).
   W. Dietsche, H. Kinder, J. Mattes, and H. Wühl, Phys. Rev. Letters 45, 1332 (1980).
- [9] J. Willer, G. Fritsch, and E. Lüscher, J. Non. Cryst. Sol. 46, 321 (1981).
- [10] G. Fritsch, J. Willer, A. Wildermuth, and E. Lüscher, to appear in J. Phys. F 12 (1982).
- [11] J. Willer, G. Fritsch, and E. Lüscher, Appl. Phys. Letters 36, 859 (1980).
- J. Jäckle and K. Froböse, J. Phys. F10, 471 (1980). Ö. Rapp, J. Jäckle, and K. Froböse, to be published in J. Phys. F.